Introduction to Quantum Teleportation

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(Dated: August 11, 2006)

What is teleportation? Roughly speaking, there is a Lab A and a Lab B, and each lab has a box. The goal of teleportation is to take any object that is placed in Box A and move it to Box B.

Of special interest to science fiction fans (among others) is human teleportation, where a brave telenaut (whom we shall call Jim) enters Box A and uses the teleportation machine to travel to Lab B.

It turns out that human teleportation appears possible in principle, though is probably impossible in practice. Nevertheless, teleportation of much smaller objects like individual spins is not only possible, but has been accomplished in the laboratory. Our goal here is to explain both how teleportation is done and why it is interesting.

The beginning and end of the following discussion are non-technical and should be accessible to most readers. In the middle we will actually describe the teleportation process in the language of quantum mechanics. Along the way we will introduce some of the basic ideas of quantum mechanics. The calculations should be accessible to those readers who feel comfortable with mathematics but have no prior knowledge of quantum mechanics.

I. CLASSICAL TELEPORTATION

Let's start by assuming that the world is perfectly classical, that is, let's not worry about the effects of quantum mechanics. Can we do teleportation?

As stated above the problem is trivial and the solution is called a truck. We load the cargo of box A onto a truck, we drive the truck over to lab B, and unload the cargo into box B. Presto exchange-o, we have teleportation!

But that is not the solution we really wanted, so let's build a wall between labs A and B. Now no trucks can get through.

Unfortunately, if this wall is perfect and separates Labs A and B into two different universes, then there is nothing that can be done to move things between the two universes and our poor telenaut Jim will be forever stuck in Lab A.

To make the problem both possible and interesting let's allow a single telephone line between universes A and B. We are now in the situation pictured in Figure I. Can we teleport Jim from A to B now?

What we are trying to build now is essentially a fax machine. A giant 3-D fax machine, but a fax machine nonetheless. Into the fax machine at A goes Jim and out of the fax machine at B we get a copy of Jim.

The first objection that you could raise is that we now have two copies of Jim, which may not be ideal. But this is an easily fixed problem. We buy a shredder and attach it to the fax machine at A so that it destroys the originals after they pass through the fax.

So we run Jim through the shredder at A and now there is only on copy at B. Will this be painful for Jim? Maybe (hence the title "brave" telenaut). But remember that the surviving copy at B was made before the "original" at A was put into the shredder. From the point of view of the copy at B, he entered the box at A and exited at B and no pain was ever felt.

A second objection is that we are only getting an approximate copy of Jim at B. Certainly a standard fax machine has a fairly poor resolution, however there is no reason why we can't build very very accurate fax machines.

Now it is true that the copy at B will never be perfect. But that shouldn't be a problem. Even if we used a truck to transport an object from A to B, the object that arrives at B would be slightly different from the one that left

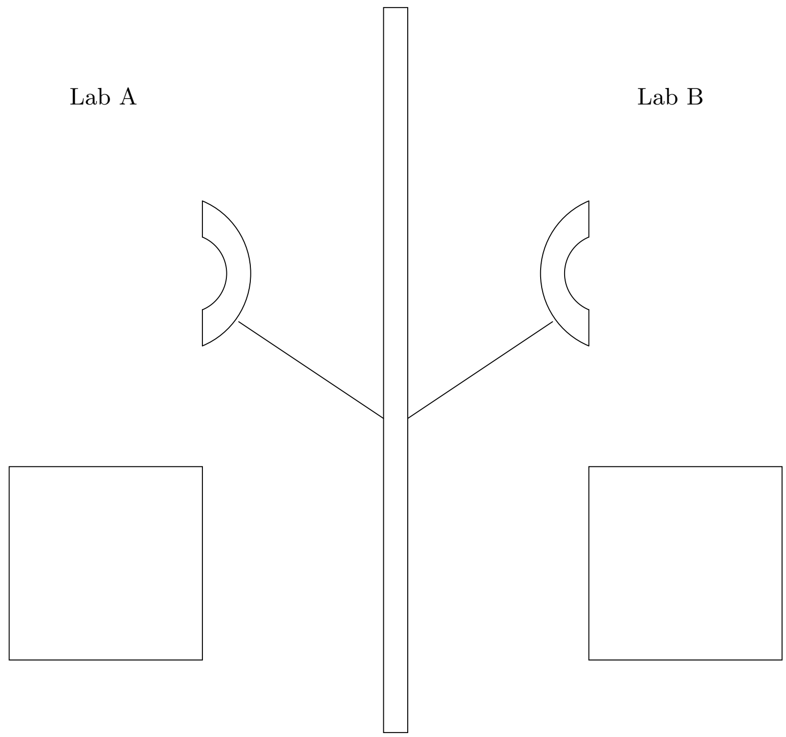

FIG. 1: The setup for teleportation

A. Along the way it will be shaken a bit or it might get hit by some cosmic rays which will change the state of a few atoms. Our goal should be that the errors that appear when we teleport Jim via the fax machine should be comparable to the changes that would have occurred when moving Jim in a truck. That is, a few very very small errors should be acceptable.

An important thing to notice is that our giant fax machine is not intended to transfer matter and energy, just like a regular fax machine would not be used to transmit blank papers. We always assume that we have the appropriate matter and energy available in Lab B and our goal is simply to assemble it into the pattern of the object placed in Box A.

So can we build a classical teleportation device as described? The answer appears to be yes. That doesn't mean that it is easy. It would be an incredible engineering feat to build a giant 3-D super-accurate fax machine. But it really is just a difficult engineering problem. From the point of view of a physicist there is no reason why this shouldn't be possible.

II. QUANTUM TELEPORTATION

But now we remember that the world is quantum mechanical, and realize that there is a problem...

What is the fax machine supposed to do?

1. Fully measures the state of the input
2. Transmits the results via the phone
3. Reconstructs the original from the received description.

Step 1 is already impossible in a quantum world because of the Heisenberg uncertainty principle. We could measure the position of all the particles forming Jim but then we wouldn't get a chance to measure the momentum of those particles. Alternatively, we could measure the momentum but then not the position. One can also envision a mixed strategy where we measure some positions and some momenta, however the uncertainty principle basically guarantees that we will never obtain enough information to rebuild even a modestly good copy of Jim.

It appears that even before running Jim through the shredder, the measurement process will likely destroy the only good copy without obtaining the required information to rebuilt Jim anew.

The surprising result of quantum teleportation is that even though the "measure and reconstruct" procedure does not work, there is an alternative procedure that effectively realizes teleportation in the quantum world.

In fact, it was not until the publication of a 1993 paper by Bennett, Brassard, Crepeau, Jozsa, Peres and Wootters that we realized quantum teleportation was possible. That is some 70 years after the formulation of the theory of quantum mechanics!

Effectively we realized that quantum teleportation, which we thought to be impossible, is only very very hard. What is the difference between the two notions? Traveling faster than the speed of light is impossible, traveling at say 99% of the speed of light is possible but very hard to do.

The upgrade in status from impossible to very very hard may not be very significant to those who would like to actually build such a device. But to a physicist it makes a world of difference, and is a very exciting discovery.

So let me begin by describing the setup for quantum teleportation, which is almost identical to the setup for classical teleportation described above. Again, we will have Labs A and B, each with a box, and we will try to move the contents of box A to box B. The two labs will be separated by a wall and only connected by a phone.

We have to be careful in specifying what kind of phone. If this phone allows sending quantum information back and forth, then the problem of quantum teleportation becomes relatively trivial. It is similar to the classical case when we allowed trucks to move objects between A and B.

The interesting case is when the phone allows only the passage of classical information. You can think of the phone as measuring all signals as they pass through the phone. All standard phones are classical phones.

In effect, what we are asking here is can we use our standard classical communication tools to transmit the state of a quantum system.

Thus far our setup for quantum teleportation is equal to the one for classical teleportation. But there is one important difference. In the quantum case, Labs A and B must begin with something called an entangled quantum state, which will be destroyed by the teleportation procedure.

Roughly speaking an entangled state is a pair of objects that are correlated in a quantum way. Below we will describe a specific example known as the "singlet state" of two spins. However, let us first explore the consequences of this extra requirement for quantum teleportation.

To prepare an entangled state of two particles, one essentially has to start with both particles in the same laboratory, let's say Lab A. Now we have the problem of sending one of the particles to Lab B. In principle, we could use quantum teleportation to send this particle to B, but this process would destroy one entangled state to create another entangled state, a net gain of zero. In any case, we have to worry about how the first entangled state is created.

The only solution is that sometime in the past the wall that separates Lab A and Lab B must not have been there. At that time the scientists from the two labs met, created a large number of entangled states, and carried them to their respective laboratories.

Think of two friends who lived nearby, but now one is moving away. They can create some entangled states that the friend who is moving can carry with him when he leaves, and then they can use those to teleport things back and forth. However, if they had never met in person and have no friends in common (who could have met with both of them) then quantum teleportation becomes impossible.

So returning to our brave telenaut Jim, he will be able to teleport to the labs of his friends. But also he could use two teleportations to travel to the labs of people whom he has never met personally, but who are friends of his friends.

Similarly, he can teleport to the labs of the friends of his friends of his friends, and so on. However, teleporting to say a distant planet or to some other place we have never had contact with is impossible.

The entanglement requirement poses a second problem, since as we mentioned above it is destroyed when used. Entanglement is effectively a resource that is slowly depleted as teleportations occur. It can be renewed by meeting in person and then carrying entanglement back from Lab A to Lab B, but it has to be transported without the use of teleportation. In principle this is difficult, otherwise we wouldn't have bothered using teleportation from A to B in the first place. However, the idea is that one difficult journey from A to B can allow in the future many quick transfers from A to B.

I should mention one last important detail of quantum teleportation. In the classical case we decided to run Jim through the shredder in Lab A after "faxing" him to lab B. But it seems like this step was optional, and we could have chosen to end up with two copies of Jim. In the quantum case this is not possible, because quantum information cannot be copied. The only way to teleport an object to Lab B is to destroy the object at Lab A.

Philosophically, one can say that if there can only ever be one copy of Jim at any time, and the copy of B survives the teleportation process in a pain free manner, then whatever is destroyed at in Lab A could not have been a copy of Jim.

However, we shall leave moral questions of this sort to the philosophers, and instead turn our attention now to the mathematics of quantum teleportation.

III. THE MATHEMATICS OF ONE QUANTUM SPIN

As most people learn from a chemistry course, electrons have spins that can be either up or down. What this means is that there exists a device called a Stern-Gerlach machine which measures the vertical angular momentum with which an electron is spinning. It turns out that the result of the measurement is always either $+1/2$ or $-1/2$ in a particular set of units. We call the first case spin up and the second spin down.

If the above seems confusing, all you need to remember is that there exists a device that sorts electrons into two categories: spin up and spin down.

To describe that state of an object that is quantum mechanical, such as spin, physicists use the funny looking symbols

$$|v\rangle$$

called kets. The important part of the above expression is the v, which can be any symbol that represents a state. The bar on the left and the greater than sign on the right have no significance beyond stating that v is a quantum state. The two states of a spin, up and down, correspond to two different kets which are written $|\uparrow\rangle$ and $|\downarrow\rangle$ respectively.

However, the rules of quantum mechanics imply that a spin can also be in a state that is a sum (often called superposition) of the spin up and spin down states. The most general state for a spin has the form

$$a|\uparrow\rangle + b|\downarrow\rangle,$$

where a and b are complex numbers.

What happens if a spin in the above state is put into a Stern-Gerlach machine? With probability $|a|^2$ we will obtain outcome $+1/2$ (that is, spin up) and with probability $|b|^2$ we will obtain outcome $-1/2$ (or spin down). The probabilities must sum to one and we have the constraint $|a|^2 + |b|^2 = 1$. The above procedure of determining whether the spin is up or down is known as a measurement in the vertical direction, and also known as a measurement in the z basis.

It is also possible to measure the spin along the horizontal direction (also known as the x basis) using a modified Stern-Gerlach machine. This will sort spins into two different groups known as spin right or spin $+1/2$ in the x direction, and spin left or spin $-1/2$ in the x direction. Spins in these states are denoted respectively by $|\rightarrow\rangle$ and $|\leftarrow\rangle$.

These new vectors are related to spin up and spin down states by the following equations

$$|\uparrow\rangle = \frac{1}{\sqrt{2}}|\rightarrow\rangle + \frac{1}{\sqrt{2}}|\leftarrow\rangle,$$
$$|\downarrow\rangle = \frac{1}{\sqrt{2}}|\rightarrow\rangle - \frac{1}{\sqrt{2}}|\leftarrow\rangle.$$

And so a state

$$\begin{aligned} a|\uparrow\rangle + b|\downarrow\rangle &= a\left(\frac{1}{\sqrt{2}}|\rightarrow\rangle + \frac{1}{\sqrt{2}}|\leftarrow\rangle\right) + b\left(\frac{1}{\sqrt{2}}|\rightarrow\rangle - \frac{1}{\sqrt{2}}|\leftarrow\rangle\right) \\ &= \left(\frac{a+b}{\sqrt{2}}\right)|\rightarrow\rangle + \left(\frac{a-b}{\sqrt{2}}\right)|\leftarrow\rangle, \end{aligned}$$

when measured in the x basis, has a probability $|\frac{a+b}{\sqrt{2}}|^2$ of being seen in the $+1/2$ or spin-right state and a probability $|\frac{a-b}{\sqrt{2}}|^2$ of being seen in the $-1/2$ or spin-left state.

In particular, if we started with the spin-up state ($a = 1$ and $b = 0$) or the spin down state ($a = 0$ and $b = 1$), then there is an equal chance of observing left or right. This is the uncertainty principle for spins.

After any measurement the state of the spin is changed to the outcome of the measurement. That is, if we measure in the x basis and observe the outcome $-1/2$, then the state of the spin will become $|\leftarrow\rangle$ irrespective of what the state of the spin was before the measurement.

There are other measurements that can be done on a spin. In particular, it can be measured along any axis. Each of these measurements may give us some information about the coefficients a and b, but never full knowledge. And since the state is effectively destroyed after the first measurement, it is impossible to ever determine the numbers a and b given only a single copy of the state. This is what we mean by the statement that we can't measure a spin and then use the results to reconstruct a new spin with the same state.

Our goal below will be to describe the teleportation of the spin of a single electron. That is, we shall place a single electron in Box A and a single electron in Box B. The goal is to make sure that the spin of the electron in Box B after teleportation is equal to the spin of the electron in Box A before teleportation. We won't care if the momentum and position (relative to the box) of the electrons are the same. We shall call this the teleportation of a spin.

It may seem like this is a much weaker goal than teleporting the full state (i.e., its position, momentum and spin) of an electron. However the techniques described below can be extended to teleport positions and momenta as well. Furthermore, it turns out that the spin is already a fairly interesting quantum mechanical object. A spin is equivalent to one qubit, which is the quantum generalization of a bit.

IV. THE MATHEMATICS OF TWO QUANTUM SPINS

When we have two spins, each of them can be individually in the up or down state, leading to the four states $|\uparrow,\uparrow\rangle$, $|\uparrow,\downarrow\rangle$, $|\downarrow,\uparrow\rangle$ and $|\downarrow,\downarrow\rangle$. The most general two-spin state is a superposition of these states:

$$a|\uparrow,\uparrow\rangle + b|\uparrow,\downarrow\rangle + c|\downarrow,\uparrow\rangle + d|\downarrow,\downarrow\rangle.$$

The coefficients have a similar interpretation as before. If both spins are measured in the vertical or z basis, then $|a|^2$ is the probability of obtaining two up outcomes, $|b|^2$ is the probability of obtaining up for the first spin and down for the second spin, and so on. Clearly we need $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$.

What happens if we just measure the first spin? By consistency we should see the up outcome with probability $|a|^2 + |b|^2$ and the down outcome with probability $|c|^2 + |d|^2$. After the measurement, the state of the two spins becomes

$$\frac{a}{\sqrt{|a|^2+|b|^2}}|\uparrow,\uparrow\rangle + \frac{b}{\sqrt{|a|^2+|b|^2}}|\uparrow,\downarrow\rangle$$

if the up outcome was obtained, and

$$\frac{c}{\sqrt{|c|^2+|d|^2}}|\downarrow,\uparrow\rangle + \frac{d}{\sqrt{|c|^2+|d|^2}}|\downarrow,\downarrow\rangle$$

if the down outcome was obtained. Basically, if we measure the first spin as being up, we remove all terms in the sum where the first spin is down, and then we multiply all the remaining coefficients by some number that makes their magnitude squared sum to one.

A second important fact about two spin states is that they are distributive in the following sense: If we know that for one spin $|\uparrow\rangle = \frac{1}{\sqrt{2}}|\rightarrow\rangle + \frac{1}{\sqrt{2}}|\leftarrow\rangle$, then the equality can be applied to the first spin to obtain equations such as

$$\begin{aligned} |\uparrow,\uparrow\rangle &= \frac{1}{\sqrt{2}}|\rightarrow,\uparrow\rangle + \frac{1}{\sqrt{2}}|\leftarrow,\uparrow\rangle, \\ |\uparrow,\downarrow\rangle &= \frac{1}{\sqrt{2}}|\rightarrow,\downarrow\rangle + \frac{1}{\sqrt{2}}|\leftarrow,\downarrow\rangle, \end{aligned}$$

and similarly it can be applied to the second spin to obtain

$$\begin{aligned} |\uparrow,\uparrow\rangle &= \frac{1}{\sqrt{2}}|\uparrow,\rightarrow\rangle + \frac{1}{\sqrt{2}}|\uparrow,\leftarrow\rangle, \\ |\downarrow,\uparrow\rangle &= \frac{1}{\sqrt{2}}|\downarrow,\rightarrow\rangle + \frac{1}{\sqrt{2}}|\downarrow,\leftarrow\rangle. \end{aligned}$$

V. THE SINGLET STATE

Now it is time to introduce one of the most important two-spin states: the singlet state, defined by

$$\frac{1}{\sqrt{2}}|\uparrow,\downarrow\rangle - \frac{1}{\sqrt{2}}|\downarrow,\uparrow\rangle.$$

The singlet state is an entangled state, that in some sense is as entangled as two spins can get. It has the the following important properties:

1. If either spin is measured along any axis, the outcome will always yield $+1/2$ with 50% probability and $-1/2$ with 50% probability.
2. If we measure one spin along any axis and then measure the other spin along the same axis the results will always be anticorrelated (that is, one spin will yield outcome $+1/2$ and the other spin will yield $-1/2$).

That the above is true for measurements along the z axis should be obvious given the rules of measurement we defined above. Checking that this is also true for measurements along the x axis is a good exercise.

The first rule basically states that we have no information about the measurement outcome of a single spin. However, the second rule essentially states that as soon as we measure one spin, we know the state of the second spin: it is a spin in a direction opposite to the first.

At first these entangled states may seem very mysterious, but something very similar can be constructed in the classical world. Ask a friend to draw either an up arrow or a down arrow on a piece of paper without telling you which he chose, and then make a copy of the paper without looking at it. Now in principle we have a very similar situation, you don't know what is drawn on either sheet of paper but as soon as you see one you know exactly what is on the other one. The fact that in the quantum case the spins are always opposite is not a significant difference either: just rotate one of the papers after the photocopying step.

What is truly special about the entangled state is that in the quantum world there are many ways of measuring a spin, as opposed to just one way of looking at a sheet of paper, and for each possible measurement of the spins there is an anticorrelation of the results.

VI. UNITARY OPERATIONS

Thus far all we have ever done with quantum systems is measure them. These measurements give us some information about the state but also have the consequence of destroying the remaining information.

There is another class of operations that reveal nothing about the quantum state and hence do not destroy any of the hidden “quantum information.” These operations are called unitary operations.

The simplest example of a unitary operation is a rotation. Say a rotation around the x axis by 180 degrees. This will change an spin up into a spin down but will leave a right-spin invariant.

The operation that rotates a spin by 180 degrees around the x axis is important enough that it is often just called the X operation or the X gate. When acting on a general state $a|\uparrow\rangle + b|\downarrow\rangle$ the X operation will transform it into the state $b|\uparrow\rangle + a|\downarrow\rangle$. We write this as

$$X\left(a|\uparrow\rangle + b|\downarrow\rangle\right) = a|\downarrow\rangle + b|\uparrow\rangle.$$

The fact that it does not destroy any quantum information follows from the fact that it can be undone, mainly by performing another rotation by 180 degrees around the x axis.

Another very important operation is a rotation by 180 degrees around the z axis, which exchanges the left and right spin states. The operation is called the Z operation or Z gate and acts on general states as

$$Z\left(a|\uparrow\rangle + b|\downarrow\rangle\right) = a|\uparrow\rangle - b|\downarrow\rangle.$$

That is, it applies a minus sign to the $|\downarrow\rangle$ state. This may seem a little strange, unfortunately we will have to defer a discussion of this minus sign to some other time (If you examine the X gate closely you will see that it applies a similar minus sign to the $|\leftarrow\rangle$ state).

There is one final operation which will be required for quantum teleportation. We call it the “Controlled-X” and sometimes abbreviated it as C-X. It acts on two spins at a time as follows

$$\text{C-}X\left(a|\uparrow,\uparrow\rangle + b|\uparrow,\downarrow\rangle + c|\downarrow,\uparrow\rangle + d|\downarrow,\downarrow\rangle\right) = a|\uparrow,\uparrow\rangle + b|\uparrow,\downarrow\rangle + c|\downarrow,\downarrow\rangle + d|\downarrow,\uparrow\rangle,$$

that is, if the first spin is down it flips the second spin but if the first spin is up then it leaves the second spin invariant.

It seems easy to do the above operation if we measure the first spin in the z basis, and then do the required operation based on the result of the measurement. It turn out that this operation can also be done without any measurements (and hence without destroying any of the quantum information) so long as both spins are in the same laboratory.

In fact, all three of the above operations X, Z and C-X can be done in practice in a laboratory, and we shall use them as the building blocks for our teleportation scheme.

VII. THE SETUP FOR QUANTUM TELEPORTATION

Now we are finally ready to return to the problem of teleportation. The setup for the teleportation experiment is as follows: We start with a spin in an unknown state $a|\uparrow\rangle + b|\downarrow\rangle$, which we are trying to teleport. By “unknown state” we simply mean that we don’t know what are the values of the coefficients a and b. This spin begins in Lab A.

We also start with an entangled “singlet state” with state $\frac{1}{\sqrt{2}}|\uparrow,\downarrow\rangle - \frac{1}{\sqrt{2}}|\downarrow,\uparrow\rangle$. This state consists of two spins the first of which is in Lab A and the second is in Lab B.

The state of all three spins can be written as

$$\frac{a}{\sqrt{2}}|\uparrow,\uparrow,\downarrow\rangle - \frac{b}{\sqrt{2}}|\downarrow,\uparrow,\downarrow\rangle - \frac{a}{\sqrt{2}}|\uparrow,\downarrow,\uparrow\rangle + \frac{b}{\sqrt{2}}|\downarrow,\downarrow,\uparrow\rangle.$$

That is, in quantum mechanics the amplitudes multiply. If the first spin had a coefficient a for the $|\uparrow\rangle$ state and the second and third spins had a coefficient $1/\sqrt{2}$ for the $|\uparrow,\downarrow\rangle$ state, then their combined state will have a coefficient of $a/\sqrt{2}$ for the state $|\uparrow,\uparrow,\downarrow\rangle$. The other coefficients can be obtained by similar arguments.

Recapping, we start with three spins:

1. Spin 1 is the unknown spin that we are trying to teleport and is located in Lab A.
2. Spin 2 is the first spin of the entangled pair and is located in Lab A.
3. Spin 3 is the second spin of the entangled pair and is located in Lab B.

At the end of the teleportation procedure spin 3 will have the state $a|\uparrow\rangle + b|\downarrow\rangle$. We still won't know the value of the coefficients a and b, but the state of spin 1, which was in Lab A will have been moved to spin 3, which is in Lab B.

VIII. A RECIPE FOR QUANTUM TELEPORTATION

Teleportation is accomplished by the following five steps:

- **Step 1:** Apply the C-X operation to spins 1 and 2.
- **Step 2:** Measure spin 2 in the z basis.
- **Step 3:** Measure spin 1 in the x basis.
- **Step 4:** Lab A calls Lab B and informs them of the outcome of the two measurements.
- **Step 5:** In Lab B they do the following:
  - If the measurement outcomes were $\downarrow$ for spin 2 and $\rightarrow$ for spin 1: Do nothing.
  - If the measurement outcomes were $\downarrow$ for spin 2 and $\leftarrow$ for spin 1: Apply the Z gate to spin 3.
  - If the measurement outcomes were $\uparrow$ for spin 2 and $\rightarrow$ for spin 1: Apply the X gate to spin 3.
  - If the measurement outcomes were $\uparrow$ for spin 2 and $\leftarrow$ for spin 1: Apply the Z gate to spin 3 and then apply the X gate to spin 3.

The first three operations can all be done in Lab A with no help from Lab B. We are allowed to do the C-X operation because both spins 1 and 2 are in Lab A. The last step can be completed entirely in Lab B, once the results of the measurements are known.

IX. PROOF OF TELEPORTATION

Starting from

$$\frac{a}{\sqrt{2}}|\uparrow,\uparrow,\downarrow\rangle - \frac{b}{\sqrt{2}}|\downarrow,\uparrow,\downarrow\rangle - \frac{a}{\sqrt{2}}|\uparrow,\downarrow,\uparrow\rangle + \frac{b}{\sqrt{2}}|\downarrow,\downarrow,\uparrow\rangle$$

we apply the C-X operation from Step 1, which produces the state

$$\frac{a}{\sqrt{2}}|\uparrow,\uparrow,\downarrow\rangle - \frac{b}{\sqrt{2}}|\downarrow,\downarrow,\downarrow\rangle - \frac{a}{\sqrt{2}}|\uparrow,\downarrow,\uparrow\rangle + \frac{b}{\sqrt{2}}|\downarrow,\uparrow,\uparrow\rangle.$$

Now we measure spin 2 in the z basis. The probability of getting spin up is $|\frac{a}{\sqrt{2}}|^2 + |\frac{b}{\sqrt{2}}|^2 = (|a|^2 + |b|^2)/2 = 1/2$ and the probability of getting spin down is exactly the same. That is, there is a 50% chance that we will see either outcome.

Let's assume that we got the outcome spin up outcome for spin 2 (the other case is similar). The state will now be

$$a|\uparrow,\uparrow,\downarrow\rangle + b|\downarrow,\uparrow,\uparrow\rangle.$$

The $1/\sqrt{2}$ are absent because after a measurement we always multiply the state by the number that makes the magnitude squared of the coefficients sum to one. In this case, that number exactly cancels the $1/\sqrt{2}$ factors.

Let's rewrite the first spin in terms of the $|{\rightarrow}\rangle$ and $|{\leftarrow}\rangle$ states as follows

$$\frac{a}{\sqrt{2}}|\uparrow,\uparrow,\downarrow\rangle + \frac{b}{\sqrt{2}}|\downarrow,\uparrow,\uparrow\rangle \;=\; a\left(\frac{1}{\sqrt{2}}|\rightarrow,\uparrow,\downarrow\rangle + \frac{1}{\sqrt{2}}|\leftarrow,\uparrow,\downarrow\rangle\right) + b\left(\frac{1}{\sqrt{2}}|\rightarrow,\uparrow,\uparrow\rangle - \frac{1}{\sqrt{2}}|\leftarrow,\uparrow,\uparrow\rangle\right).$$

Now we can do Step 3, where we measure the first spin in the x basis. Again we obtain the spin right outcome with probability $|\frac{a}{\sqrt{2}}|^2 + |\frac{b}{\sqrt{2}}|^2 = (|a|^2 + |b|^2)/2 = 1/2$ and similarly for the spin left outcome.

Let's assume we got the spin right outcome for spin 1 (again the other case is similar), and therefore the state becomes

$$a|\rightarrow,\uparrow,\downarrow\rangle + b|\rightarrow,\uparrow,\uparrow\rangle,$$

where again the factors of $1/\sqrt{2}$ disappear as before.

In step 4 Lab A communicates the results to Lab B. Finally, in Step 5 Lab B must perform the X gate to spin 3, which changes the state into

$$a|\rightarrow,\uparrow,\uparrow\rangle + b|\rightarrow,\uparrow,\downarrow\rangle.$$

This is our goal. The expression says that the state for the two spins in Lab A is $|\rightarrow,\uparrow\rangle$, whereas the state for the spin in Lab B is $a|\uparrow\rangle + b|\downarrow\rangle$.

X. CAN QUANTUM TELEPORTATION BE USED FOR SUPERLUMINAL COMMUNICATION?

If we tried to define a colloquial notion of teleportation it would probably have two main properties: That objects move from A to B without "passing" through the space in between and that it be done instantaneously, or at least very very fast.

Roughly speaking, our teleportation schemes satisfy the first property. However, thus far we haven't discussed the speed at which teleportation should occur.

Teleportation as defined here requires sending a message from Lab A to Lab B using a regular phone. The message will travel at the speed of light from A to B. Therefore, our version of teleportation cannot be instantaneous and does not allow for travel faster than the speed of light.

In fact, teleportation might be significantly slower than light travel if the measurement and reconstruction procedures are slow. However, if we are teleporting a person (or some other system that is not static) then the measurement and reconstruction procedures need to be performed nearly instantaneously. After all, if you get to see as your feet are slowly measured and disassembled, the process would likely not be pain-free.

At first glance, though, there seems to be a way to use the teleportation procedure for superluminal communication. That is, by measuring the spins in Lab A, we are somehow instantaneously modifying the spin in Lab B. Whether or not this is a good description of what is going on depends which interpretation of quantum mechanics is used to describe the system (there are actually many interpretations of quantum mechanics which describe the above process in very different ways). However, all interpretations of quantum mechanics agree on one fact: that such tricks cannot be used for superluminal communication.

The basic idea of such a proof is to check that, when averaged over all the outcomes obtained in Lab A, any measurement done in Lab B will always yield 50-50 outcomes, no matter what state is being teleported. Therefore the measurements in Lab B cannot convey any useful information, at least until such a time when the correction operators have been applied.

Unfortunately all modern theories of physics predict that both faster than light travel and faster than light communication are impossible.

XI. REAL EXPERIMENTS THAT DO TELEPORTATION

A number of groups conducted experimental realizations of the quantum teleportation procedure described above in the years 1997 and 1998, using a variety of different systems such as the spin (or polarization) of photons and the spin of atoms. In many cases Labs A and B were the left and right side of a table, and the spins were teleported roughly 50 cm.

The reason distance becomes relevant has to do with the distribution of entanglement which becomes harder as the separation between the two "labs" increases. A second related problem is the storing of entanglement which can only be done for very short periods, so in practice most early experiments distribute the entanglement only moments before it is to be used for teleportation. However, these experiments were sufficient to convince most physicists that teleportation of spins is possible.

Since 1997 there have also been many improved versions of the teleportation experiment. For instance, the distance has been increased in one experiment to 600 m, and the accuracy of the teleported state has also been slowly improving.

However, at the time this document was written, most experiments have only teleported a single spin. In principle, if you can teleport one spin, then you can teleport many spins simply by repeating the experiment in series many times. But this roughly only works on disjoint spins. To teleport a single object comprised of many spins is still out of reach of present day experiments.

In the future, though, we should see experiments that teleport large numbers of spins. Certainly, if a practical quantum computer is ever built then the same technology would likely allow us to teleport a few thousand spins. It is likely that this will happen within the next 30–50 years, if not sooner.

XII. BUT WILL WE EVER BE ABLE TO TELEPORT PEOPLE?

There are some 10^{29} matter particles comprising a human person, each of which has position and momentum degrees of freedom in addition to spin. In principle, we might also need to teleport the photons, gluons and other energy particles comprising a person. Teleporting all that is going to be significantly harder than a few thousand spins. It is probably a good guess that teleportation of humans will never be possible.

Are we at least sure that it is possible to teleport humans in principle?

While most scientists expect that ten, hundreds and maybe even thousands of spins will be teleported in practice some day, the teleportation of a human being, even in principle, is actually still a controversial subject.

I would roughly divide people into three schools of thought.

The first group of physicists would argue that there is a soul, consciousness or spirit that permeates the human body that cannot be described by science. Unfortunately, in this view by definition we are prevented from using science to determine if teleportation is feasible.

A second group of physicists would disagree with human teleportation because of something known as the measurement problem. Roughly speaking, any object that is capable of performing quantum measurements cannot itself be a quantum object, and therefore cannot be teleported using quantum teleportation. In this view, small numbers of particles are quantum but at some point when you combine enough particles you end up with a classical or "observer" object, which cannot be described by the laws of quantum mechanics.

In principle, such a belief will have experimental consequences, as we should be able to determine at what point do objects stop being quantum mechanical. At the moment there is neither any experimental evidence for such observer objects nor even a consistent theory that could describe them. On the other hand, it is also true that presently it is very hard to experimentally study large quantum systems, and so it is quite possible that something interesting will happen when a large enough system is examined.

The third school of though (which I am partial to) would say that all objects big and small are quantum mechanical, and therefore in principle can be teleported. What happened with the measurement problem? I would argue that measurements never actually occur. What happens is that the observer becomes entangled with the system he is measuring, and this appears to the observer as if a measurement was performed. The mathematics for this process works out quite nicely, but it does leave the nagging question of why does it feel like we are constantly measuring the world?

Of course, the final answer to whether teleportation of people is possible even in principle must wait for the formulation of a complete theory of physics, one which unifies relativity with quantum mechanics.

In the meantime, one can ask if there any applications for teleporting thousands of spins?

The answer is probably yes. In the future it is likely that quantum computers (i.e., computers capable of processing quantum information) will be built and may even be as ubiquitous as classical computers are today. These computers will need to exchange quantum information. One way these exchanges of information can occur is via a quantum phone, that is, a device capable of sending and received quantum messages. But when such phones are not available, the alternative is to do teleportation using a regular phone. So don't be surprised if some day in the next 100 years you see a quantum teleportation device for sale in your local computer store.